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Dilatonic Dark Energy Model with Late Time de Sitter Attractor

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Based on Weyl-scaled induced gravitational theory, we regard dilaton field in this theory as a candidate of dark energy. We construct a dilatonic dark energy model and its phantom model, that admit late time de Sitter attractor solution. When we take the potential of dilaton field as the form $\lambda e^{\beta\sigma^2}$ which has been studied in supergravity model and the famous Mexican hat potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$, we show mathematically that these attractor solutions correspond to an equation of state $\omega = -1$ and a cosmic density parameter $\Omega_\sigma = 1$, which are important features for a dark energy model that can meet the current observations.

KEY WORDS: dilaton; dark energy; Weyl-scaled; Mexican hat potential; attractor.

1. INTRODUCTION

Recent observations from WMAP (Bennett *et al.*, 2003) suggest that our universe is dominated by a kind of dark energy which is responsible for an accelerated expansion supported by the data of type Ia supernov(Riess, 1998). The dark energy with negative pressure is two thirds of the total energy of the universe. Its state parameter ω is about equal to -1 and may even be smaller than -1(Caldwell, 2002). To explain the accelerated expansion of the universe from fundamental physics is now a great challenge. Many models have been proposed so far to fit these observations. Among these models, the most typical ones are cosmological construct and a time varying scalar field with positive or negative kinetic energy evolving in a specific potential, referred to as "quintessence" (Armendariz-Picon *et al.*, 1999; Bagla *et al.*, 2003; Elizalde *et al.*, 2004; Feinstein, 2002; Frolov *et al.*, 2002; Mazumadar *et al.*, 2001; Padmanabhan, 2002; Padmanabhan and Choudhury, 2002; Sen, 2002; Wetterich, 1998) or "phantom" (Carroll *et al.*, 2003; Chiba *et al.*, 2000; Gibbons, 2003; Li and Hao, 2004; Singh *et al.*, 2003). Further models, such as (general)Chaplygin gas (Lu *et al.*, 2005; Mak and Harko, 2005) and K-

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essence (Armendariz-Picon *et al.*, 1999; Bagla *et al.*, 2003; Elizalde *et al.*, 2004; Feinstein, 2002; Frolov *et al.*, 2002; Mazumadar *et al.*, 2001; Padmanabhan, 2002; Padmanabhan and Choudhury, 2002; Sen, 2002; Wetterich, 1998) are proposed to account for both dark energy and dark matter in a unified way. Some researchers resort to scalar-tensor gravitational theory (Banerjee and Pavon, 2002; Jordan, 1947), which is a modified relativistic theory of gravitation apparently compatible with Mach's principle. The scalar field (dilaton) in Brans-Dicke theory is then expected to account for the desired quintessence or K-essence. The Brans-Dicke Lagrangian admits two space-time metrics, the Jordan metric (Jordan, 1947) and the Pauli metric (Pauli and Fierz, 1956), which can actually describe two different physics. In Jordan-Brans-Dicke theory(JBD) and Einstein-Brans-Dicke theory (EBD), we treat respectively the Jordan and Pauli metric as physical. Cho (Cho, 1992) pointed out that only the Pauli metric can represent the massless spin-two graviton, and thus can correctly describe Einstein's theory of gravitation. This means that, as long as one wants to interpret the theory as a generalization of Einstein' theory, one must treat the Pauli metric as physical. In the Kaluza-klein theory he arrives essentially at the same conclusion, but for a different reason. One cannot treat the Jordan metric as physical, because it violates the positivity of the Hamiltonian. At the same time one must accept the Pauli metric as physical, as long as one wishes to achieve the unification of Einstein's gravitation with other interactions from the Kaluza-Klein theory. Some researchers have studied the evolution of universe in EBD (Damour and Nordtvedt, 1993). The EBD with dilatonic potential is the Weyl-scaled induced gravitational theory.

In this paper, we construct dilatonic dark energy model and its phantom model based on Weyl-scaled induced gravitational theory. There are many motivations that make us to consider the Weyl-scaled induced gravitational theory. First, the dilaton is an essential element of string theories and the low-energy string effective action (Polchinski, 1998). Second, dimensional reduction of Kaluza-Klein theories may naturally lead to the Weyl-scaled induced gravitational theory. Third , dilatonic gravities are expected to have such important cosmological applications as in the case of (hyper) extended inflation (Kolb *et al.*, 1990). Fourth, in tachyon model, the role of tachyon field in string theory in cosmology has been widely studied. However, the model of inflation with a single tachyon field generates larger anisotropy and has troubles in describing the formation of the universe. The extended inflation(shortened EI) model resolves cosmological puzzles and predicts the values of spectral indies close to observations. The process of EI can also occur in Weyl-Scaled induced gravitational theory. The success of EI model leads us to consider the dilaton field in Weyl-scaled induced gravitational theory as theoretic models of dark energy. Only a few authors have considered this theory in an accelerated universe (Torres, 2002). Reference (Torres, 2002) shows that a stage of super-expansion of unverse is possible within the induced gravitational theory with an inverse square potential.

The goal of this paper is to construct a dilatonic dark energy model and its phantom model. We would also investigate the existence and stability of de Sitter solutions in the condition of different potentials, and shall show the dependence of stable solution on the initial conditions. We numerically investigate the model with different potentials such as $Ae^{\beta\sigma^2}$ from the supergravity model (Brax and Martin, 1999) and the famous Mexican hat potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + V_0$ (Hao and Li, 2003), and then the attractor behavior is manifest.

This paper is organized as follows: In Section 2, we introduce the field equations of Weyl-scaled induced gravitational theory. In Section 3, we construct the dilatonic dark energy model and deduce the basic field equations. In Section 4, we study two special examples for the potential of dilaton and show the evolutive behavior of the universe as a dynamic system numerically. In Section 5, we consider the phantom dilatonic dark energy model-with negative kinetic energy. Section 6 is summery.

2. COSMOLOGICAL DYNAMICS IN THE PRESENCE OF DILATON FIELD

To deduce the field equation of induced gravitational theory, let us consider the action of Jordan-Brans-Dicke theory firstly

$$S = \int d^4X\sqrt{-\gamma}\left[\phi\tilde{R} - \omega\gamma^{\mu\nu}\frac{\partial_\mu\phi\partial_\nu\phi}{\phi} - \Lambda(\phi) + \tilde{L}_{fluid}(\psi)\right] \tag{1}$$

where the lagrangian density of cosmic fluid $\tilde{L}_{fluid}(\psi) = \frac{1}{2}\gamma^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - V(\psi)$, γ is the determinant of $\gamma_{\mu\nu}$ which is Jordan metric, ω is the dimensionless coupling parameter, R is the contracted $R_{\mu\nu}$. The metric sign convention is$(-, +, +, +)$. The quantity $\Lambda(\phi)$ is a nontrivial potential of ϕ field. When $\Lambda(\phi) \neq 0$ the action of equation (1) describes the induced gravity. The energy density of cosmic fluid $\widetilde{\rho} = \frac{1}{2}(\frac{d\psi}{dt})^2 + V(\psi)$ and the pressure $\widetilde{p} = \frac{1}{2}(\frac{d\psi}{dt})^2 - V(\psi)$.

However it is often useful to write the action in terms of the conformally related Einstein metric. We introduce the dilaton field σ and conformal transformation as follows

$$\phi = \frac{1}{2}e^{\alpha\sigma} \tag{2}$$

$$\gamma_{\mu\nu} = e^{-\alpha\sigma}g_{\mu\nu} \tag{3}$$

where $\alpha^2 = \frac{\kappa^2}{2\omega+3}$, $g_{\mu\nu}$ is the Pauli metric. $\kappa^2 = 8\pi G$ is taken to be one for convenience.

The action (1) becomes Eq. (4) by performing the conformal transformation equation (2) and Eq. (3)

$$S = \int d^4X\sqrt{-g}\left[\frac{1}{2}R(g_{\mu\nu}) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - W(\sigma) + L_{fluid}(\psi)\right] \tag{4}$$

where

$$L_{fluid}(\psi) = \frac{1}{2}g^{\mu\nu}e^{-\alpha\sigma}\partial_\mu\psi\partial_\nu\psi - e^{-2\alpha\sigma}V(\psi)$$

The transformation Eqs. (2) and (3) are well defined for some ω as $-\frac{3}{2} < \omega < \infty$. The conventional Einstein gravity limit occurs as $\sigma \to 0$ for an arbitrary ω or $\omega \to \infty$ with an arbitrary σ.

The nontrivial potential of the σ field, $W(\sigma)$ can be a metric scale form of $\Lambda(\phi)$. Otherwise, one can start from equation (4), and define $W(\sigma)$ as an arbitrary nontrivial potential. $g_{\mu\nu}$ is the pauli metric. Damour and Cho *et.al* pointed out that the pauli metric can represent the massless spin-two graviton in induced gravitational theory. Cho also pointed out that in the compactification of Kaluza-Klein theory, the physical metric must be identified as the pauli metric because of the the wrong sign of the kinetic energy term of the scalar field in the Jordan frame. The dilaton field appears in string theory naturally.

By varying the action Eq. (4), one can obtain the field equations of Weyl-scaled induced gravitational theory.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{3}\left\{\left[\partial_\mu\sigma\partial_\nu\sigma - \frac{1}{2}g_{\mu\nu}\partial_\rho\sigma\partial^\rho\sigma\right] - g_{\mu\nu}W(\sigma)\right.$$
$$\left. + e^{-\alpha\sigma}\left[\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}g_{\mu\nu}\partial_\rho\psi\partial^\rho\psi\right] - g_{\mu\nu}e^{-2\alpha\sigma}V(\psi)\right\} \quad (5)$$

$$\Delta\sigma = \frac{dW(\sigma)}{d\sigma} - \frac{\alpha}{2}e^{-2\alpha\sigma}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - 2\alpha e^{-2\alpha\sigma}V(\psi) \quad (6)$$

$$\Delta\psi = -\alpha g_{\mu\nu}\partial_\mu\psi\partial_\nu\sigma + e^{-\alpha\sigma}\frac{dV(\psi)}{d\psi} \quad (7)$$

The energy-momentum tensor $T_{\mu\nu}$ of cosmic fluid is

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu} \quad (8)$$

where the density of energy

$$\rho = \frac{1}{2}\dot{\psi}^2 + e^{-\alpha\sigma}V(\psi) \quad (9)$$

the pressure

$$p = \frac{1}{2}\dot{\psi}^2 - e^{-\alpha\sigma}V(\psi) \quad (10)$$

ρ and p are related to their directly measurable counterparts by $\rho = e^{-\alpha\sigma}\widetilde{\rho}$, $p = e^{-\alpha\sigma}\widetilde{p}$. We work in R-W metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \quad (11)$$

and we consider that ρ, p and σ depend only on time. So, according to Eqs. (6)–(12), we can obtain

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\sigma}^2 + W(\sigma) + e^{-\alpha\sigma}\rho\right] \tag{12}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dW}{d\sigma} = \frac{1}{2}\alpha e^{-\alpha\sigma}(\rho - 3p) \tag{13}$$

$$\dot{\rho} + 3H(\rho + p) = \frac{1}{2}\alpha\dot{\sigma}(\rho + 3p) \tag{14}$$

where ρ and p are the density and pressure of cosmic fluid, H is Hubble parameter.

3. DILATONIC DARK ENERGY MODEL

In dilaton dominated epoch, the gravitational field equations become

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\sigma}^2 + W(\sigma)\right] \tag{15}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dW}{d\sigma} = 0 \tag{16}$$

The effective density ρ_σ and effective pressure p_σ can be expressed as follows

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + W(\sigma) \tag{17}$$

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 - W(\sigma) \tag{18}$$

So, the equation of state of dilaton field is

$$\omega_\sigma = \frac{\frac{1}{2}\dot{\sigma}^2 - W(\sigma)}{\frac{1}{2}\dot{\sigma}^2 + W(\sigma)} \tag{19}$$

In order to gain more insights into the dynamical system, we introduce the new dimensionless variables

$$X = \frac{\sigma}{\sigma_0}$$

$$Y = \frac{\dot{\sigma}}{\sigma_0^2}$$

$$N = ln\ a \tag{20}$$

Equation (19) becomes

$$\omega_\sigma = \frac{\frac{\sigma_0^4 Y^2}{2} - W(X)}{\frac{\sigma_0^4 Y^2}{2} + W(X)} \tag{21}$$

The field Eqs. (15) and (16) could be rewritten as follows

$$\frac{dX}{dN} = \frac{\sigma_0 Y}{H} \tag{22}$$

$$\frac{dY}{dN} = -3Y - \frac{W'(X)}{\sigma_0^2 H} \tag{23}$$

where the prime denotes the derivative with respect to X and H could be expressed as

$$H = \left\{ \frac{1}{3H_i^2} \left[\frac{\sigma_0^4 Y^2}{2} + W(X) \right] + \Omega_{M,i} e^{-3N} + \Omega_{r,i} e^{-4N} \right\}^{\frac{1}{2}} \tag{24}$$

where H_i denotes the Hubble parameter at an initial time, $\Omega_{M,i}$ and $\Omega_{r,i}$ denote the cosmic density parameters for matter and radiation at the initial time. The initial scale factor a_i is taken to 1. At late time, N will become very large and the contributions from matter and radiation in Eq. (24) become negligible compared with dilaton field. So, H in dilaton dominated epoch can be obtained

$$H_\sigma = \left\{ \frac{1}{3} \left[\frac{\sigma_0^4 Y^2}{2} + W(X) \right] \right\}^{\frac{1}{2}}. \tag{25}$$

Taking this equation into Eqs. (21) and (22), we have

$$\frac{dX}{dN} = \sqrt{3} \frac{\sigma_0 Y}{\sqrt{\frac{\sigma_0^4 Y^2}{2} + W(X)}} \tag{26}$$

$$\frac{dY}{dN} = -3Y - \sqrt{3} \frac{W'(X)}{\sigma_0^3 \sqrt{\frac{\sigma_0^4 Y^2}{2} + W(X)}} \tag{27}$$

Equations (26) and (27) and their initial conditions determine the evolution of the dynamic system and the behaviors of the late time de Sitter attractor, which include the existence and the stability of the late time de Sitter attractor. The critical point of the above autonomous system is $(X_c, 0)$, where X_c is defined by $W'(X_c) = 0$. The cosmic density parameter for the dark energy is

$$\Omega_\sigma = \frac{\kappa \left[\frac{\sigma_0^4 Y^2}{2} \right] + W(X)}{3H^2} \tag{28}$$

Note that the energy density of the dilaton field at the critical point is $W(X_c)$ and should not vanish, thus the sufficient condition for the existence of a viable cosmological model with a late time de Sitter attractor solution should be that: the potential of the field has non-vanishing minimum(maximum for phantom) value.

The cosmic density for the radiation and matter are respective

$$\Omega_r = \frac{\Omega_{r,i} e^{-3N}}{3H^2} \tag{29}$$

$$\Omega_m = \frac{\Omega_{r,i} e^{-4N}}{3H^2} \tag{30}$$

Clearly, from Eqs. (21) and (28) one can find that $\omega_\sigma = -1$ and $\Omega_\sigma = 1$ at late time attractor. In next part, we will take different potentials $W(X)$ in Eqs. (26) and (27), and show mathematically that the evolutions of the components of cosmic density, the evolution of the parameter of state equation and the evolution of X, Y with N.

4. SPECIAL EXAMPLES

$$\mathbf{A}. W(\sigma) = \lambda e^{\beta\sigma^2} (\lambda > 0, \beta > 0)$$

The potential $W(\sigma) = \lambda e^{\beta\sigma^2}$ drives from the potential $W(\sigma) = \frac{\Lambda^{4+\alpha}}{\sigma^\alpha} e^{\frac{k}{2}\sigma^2}$ studied in the supergravity model (Brax and Martin, 1999). In reference (Brax and Martin, 1999), Brax and Martin have studied the potential $W(\sigma) = \frac{\Lambda^{4+\alpha}}{\sigma^\alpha} e^{\frac{k}{2}\sigma^2}$ model and investigated the properties of this potential in the supergravity. Based on the Kähler potential, they use string-inspired models with an anomalous $U(1)_X$ gauge symmetry and consider the the case of type I string theories, so, they conclude that the scalar potential deduced from the supergravity model has the form $W(\sigma) = \frac{\Lambda^{4+\alpha}}{\sigma^\alpha} e^{\frac{k}{2}\sigma^2}$. When the parameter $\alpha = 0$, the above potential becomes our potential. So our potential $W(\sigma) = \lambda e^{\beta\sigma^2}$ is a special example of $W(\sigma) = \frac{\Lambda^{4+\alpha}}{\sigma^\alpha} e^{\frac{k}{2}\sigma^2}$.

When the potential $W(\sigma)$ is taken as the form $\lambda e^{\beta\sigma^2}$, according to Eqs. (20), (21) and (26)–(28) become

$$\omega = \frac{\frac{1}{2}\sigma_0^4 Y^2 - \lambda e^{\beta\sigma_0^2 X^2}}{\frac{1}{2}\sigma_0^4 Y^2 + \lambda e^{\beta\sigma_0^2 X^2}} \tag{31}$$

$$\frac{dX}{dN} = \sqrt{3} \frac{\sigma_0 Y}{\sqrt{\frac{\sigma_0^4 Y^2}{2} + \lambda e^{\sigma_0^2 X^2}}} \tag{32}$$

$$\frac{dY}{dN} = -3Y - \sqrt{3} \frac{2\lambda\beta x e^{\beta\sigma_0^2 X^2}}{\sigma_0 \sqrt{\frac{\sigma_0^4 Y^2}{2} + \lambda e^{\beta\sigma_0^2 X^2}}} \tag{33}$$

$$\Omega_\sigma = \frac{\left[\frac{\sigma_0^4 Y^2}{2} + \lambda e^{\beta\sigma_0^2 X^2}\right]}{3H^2} \tag{34}$$

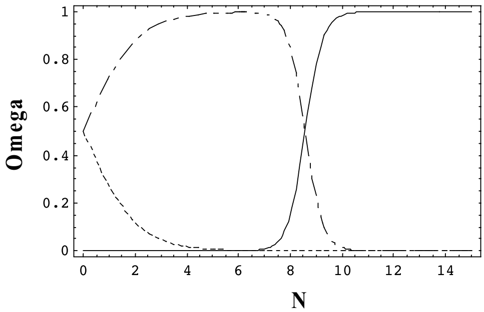

Fig. 1. The evolution of the components of cosmic density Ω_σ (solid line), Ω_m (dash-dot line), Ω_r (*dot line*) with respect to N in potential $\lambda e^{\beta\sigma^2}$.

We obtain that the potential of dilaton field has non-vanishing minimum value λ, so, there exists an attractor in this dynamic system. Based on the above equations about the dynamic system, we show their some interesting features numerically from Figs. 1–3.

$$\mathbf{B.} W(\sigma) = \frac{\lambda}{4}(\sigma^2 - \varepsilon^2)^2 + W_0$$

This potential is the famous Mexican hat potential which has been widely studied in symmetry breaking problem of unification theory. Hao and Li (2003) have considered this potential for scalar field dark energy model in cosmology. Here we take this potential as the dilaton potential in dark energy model. When the potential $W(\sigma)$ is taken as the form $\frac{\mu}{4}(\sigma^2 - \varepsilon^2)^2 + W_0$, according to

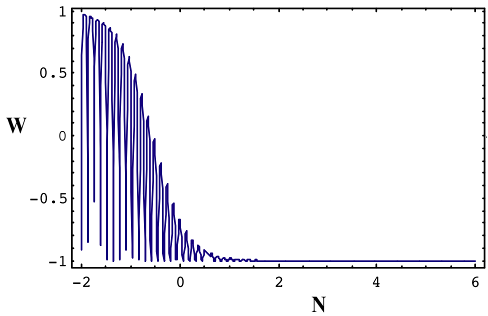

Fig. 2. The evolution of the parameter of state equation ω with respect to N in potential $\lambda e^{\beta\sigma^2}$.

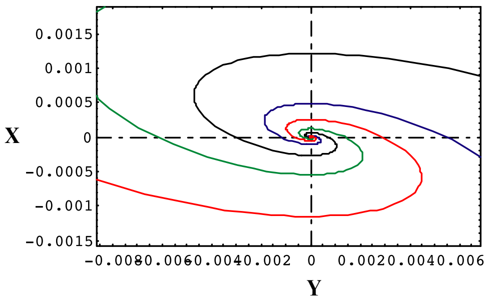

Fig. 3. Attractor properties of the system in the phase plane in potential $\lambda e^{\beta\sigma^2}$ for different initial X and Y when dilaton field is dominant.

Eqs. (20), (21) and (26)–(28) become

$$\omega = \frac{\frac{1}{2}\sigma_0^4 Y^2 - \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 - W_0}{\frac{1}{2}\sigma_0^4 Y^2 + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0} \tag{35}$$

$$\frac{dX}{dN} = \sqrt{3}\frac{\sigma_0 Y}{\sqrt{\frac{\sigma_0^4 Y^2}{2} + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0}} \tag{36}$$

$$\frac{dY}{dN} = -3Y - \sqrt{3}\frac{\lambda X\left(\sigma_0^2 X^2 - \varepsilon^2\right)}{\sigma_0\sqrt{\frac{\sigma_0^4 Y^2}{2} + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0}} \tag{37}$$

$$\Omega_\sigma = \frac{\left[\frac{\sigma_0^4 Y^2}{2} + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0\right]}{3H^2} \tag{38}$$

We obtain that the potential of dilaton field has non-vanishing minimum value W_0, so, there exists an attractor in this dynamic system. Based on the above equations about the dynamic system, we show their some interesting features numerically from Figs. 4–6.

From Figs. 1–6, we can easily find that the two systems admit attractor solutions. The state parameter ω starts from nearly 1, then quickly evolves to the regime of greater than -1, turns back to execute the damped oscillation, and reaches to -1 permanently, which is mimic the de Sitter-like behavior($\omega_\sigma = -1$). According to Figs. 3 and 6, we know that, once the sufficient condition that the potential of dilaton field has non-vanishing minimum value is satisfied, the dynamic system will admit an attractor.

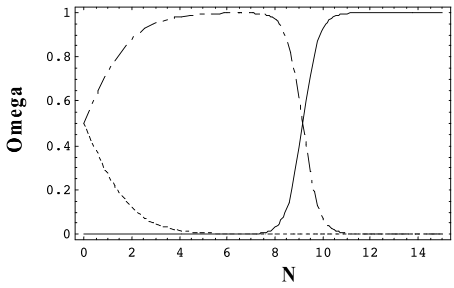

Fig. 4. The evolution of the components of cosmic density Ω_σ(solid line), Ω_m(dash-dot line), Ω_r(*dot line*) with respect to N in potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$.

5. PHANTOM MODEL

Now, we consider phantom field with negative kinetic energy. At dilaton dominated epoch, the equation of motion for dilatonic phantom field becomes

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{dW}{d\sigma} = 0 \tag{39}$$

When we take the potential $W(\sigma)$ as the Mexican hat potential $\frac{\lambda}{4}(\sigma^2 - \varepsilon^2)^2 + W_0$, we have

$$\omega = \frac{-\frac{1}{2}\sigma_0^4 Y^2 - \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 - W_0}{-\frac{1}{2}\sigma_0^4 Y^2 + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0} \tag{40}$$

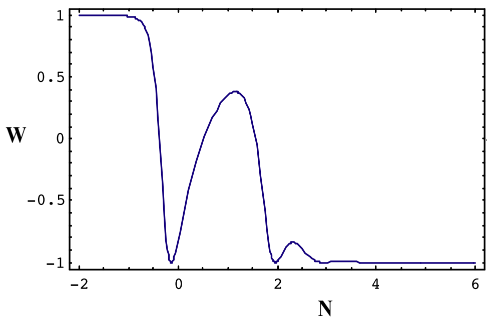

Fig. 5. The evolution of the parameter of state equation ω with respect to N in potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$.

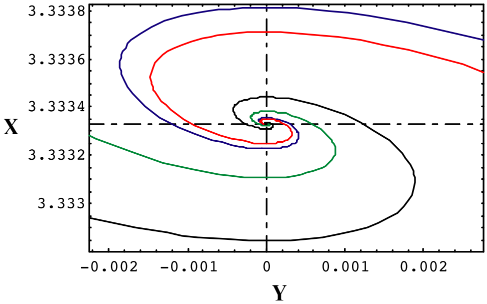

Fig. 6. Attractor properties of the system in the phase plane in potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$ for different initial X and Y when dilaton field is dominant.

$$\frac{dX}{dN} = \sqrt{3}\frac{\sigma_0 Y}{\sqrt{-\frac{\sigma_0^4 Y^2}{2} + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0}} \tag{41}$$

$$\frac{dY}{dN} = -3Y + \sqrt{3}\frac{\lambda X\left(\sigma_0^2 X^2 - \varepsilon^2\right)}{\sigma_0\sqrt{-\frac{\sigma_0^4 Y^2}{2} + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0}} \tag{42}$$

$$\Omega_\sigma = \frac{\left[-\frac{\sigma_0^4 Y^2}{2} + \frac{\lambda}{4}\left(\sigma_0^2 X^2 - \varepsilon^2\right)^2 + W_0\right]}{3H^2} \tag{43}$$

We obtain that the potential of phantom field has non-vanishing maximum value $\frac{\lambda\gamma_0^4}{4} + W_0$, so, there exists an attractor in this dynamic system. Based on the above equations about the dynamic of the system, we show their some interesting features numerically from Figs. 7–9.

From Figs 7–9, we can easily find that the phantom system admits an attractor solution. We see that the state parameter ω reaches to -1 eventually after a series of damped oscillations, which is mimic the de Sitter-like behavior ($\omega_\sigma = -1$). Due to the unusual physical properties in phantom model, the dynamic system will be stable when the energy of the dynamic system reaches the maximum value. As expected, the phantom field settles on the top of the potential permanently.

6. SUMMARY

In this paper, we construct a dilatonic dark energy model and its phantom model, with late time de Sitter attractor. We investigate the existence and stability of attractor. When we take the potential of dilaton as the form $Ae^{\beta\sigma^2}$ and $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$, we showed mathematically that the minimum(maximum for phantom model) of potential corresponding to a cosmological de Sitter phase is a dynamical

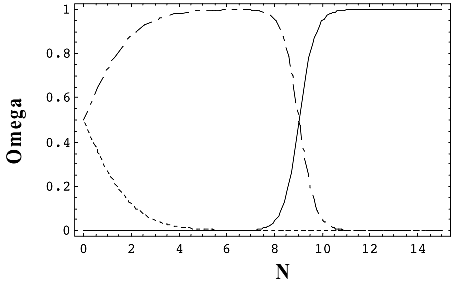

Fig. 7. The evolution of the components of cosmic density Ω_σ(solid line), Ω_m(dash-dot line), Ω_r(*dot line*) with respect to N in potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$.

attractor, and the attractor solution corresponds an equation of state $\omega = -1$ and a cosmic density parameter $\Omega_\sigma = 1$, which are important features for a dark energy model that can meet the current observations. We see from Figs. 1 and 4 that at earlier epoch of universe, the dilaton energy density can be negligible compared with radiation and matter, but, in a very recent epoch, dilaton dominated the universe little by little. Based on the above analysis, we know that the dilaton as dark energy fits the observations well. Figures 3 and 6 show that, when we set different initial conditions, the model will always tend to be stable. The range of initial conditions is wide and it alleviates the fine tuning problems. We can easily see that our current universe is not at the attractor, instead, it is just on the way to the attractor.

In this paper, the phantom model is also investigated. We study the attractor behavior via phase plane analysis, without the presence of radiation and matter. The result shows the universe will evolve to a de Sitter like attractor regime in the

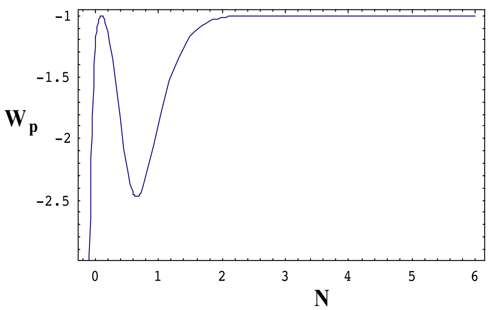

Fig. 8. The evolution of the parameter of state equation ω with respect to N in potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$.

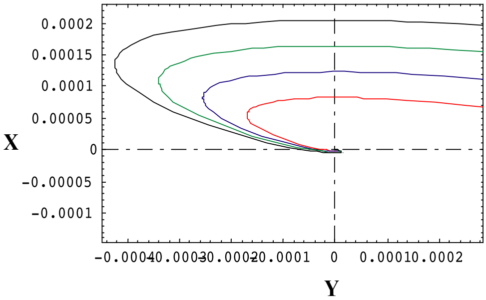

Fig. 9. Attractor properties of the system in the phase plane in potential $\frac{\lambda}{4}(\sigma^2 - \gamma_0^2)^2 + W_0$ for different initial X and Y when phantom field is dominant.

future and the phantom field energy density will dominate the universe. Theoretical cosmology with phantom models has become an active area of theoretical research. The idea of phantom cosmology is pretty new, and there are many questions remaining open yet.

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